Exam for the first semester of the year 2022/2023

Date:21/1/2023 | The band: Third, Physics

Time: 2 hours | Course: Mathematical methods | Sohag University

Course code: Math 331

Mathematics Department | Degree: 50 | Faculty of Science

Answer the following multiple-choice questions on the answer form provided

(1) The value of the integral $\int_{-\infty}^{\infty} e^{-x^2} dx$ using the gamma and beta function

(a) $\sqrt{\pi}$	(b) $\sqrt{2\pi}$	(c) $\sqrt{3\pi}$	(d) Otherwise

(2) Through the Fourier series of the function $f(x)=x^2,\ 0\le x\le 2\pi$ the value of a_0 is

(a) $\frac{2\pi^2}{3}$	(b) $\frac{4\pi^2}{3}$	(c) $\frac{8\pi^2}{3}$	(d) Otherwise

(3) Fourier transform of the function e^{-x^2} is

(a) $\frac{e^{\frac{\xi^2}{4}}}{\sqrt{2}}$	(b) $\frac{e^{-\frac{\xi^2}{4}}}{\sqrt{2}}$	(c) $-\frac{e^{\frac{\xi^2}{4}}}{\sqrt{2}}$	(d) Otherwise

(4) The Fourier transform on both sides of the heat conductive equation $\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}$ is

(a) $\frac{du_1(t)}{dt}=-k\xi^2 u_1(t)$	(b) $\frac{du_1(t)}{dt}=-k\xi^2 u_1(x)$	(c) $\frac{du_1(t)}{dt}=k\xi^2 u_1(t)$	(d) Otherwise

(5) The Laplace transform of the partial derivatives of the function $\frac{\partial^2 u}{\partial t^2}$ is

(a) $su_1(x)-u_t(x,o)$	(b) $s^2u_1(x)-su(x,0)+u_t(x,o)$	(c) $s^2u_1(x)-su(x,0)-u_t(x,o)$	(d) Otherwise

(6) Laplace transform of the function $e^{-bt}\sin at$ is

(a) $\frac{a}{(s+b)^2-a^2}$	(b) $\frac{a}{(s-b)^2+a^2}$	(c) $\frac{a}{(s+b)+a^2}$	(d) Otherwise

(7) Inverse Laplace Transform of the function $\frac{5s+4}{s^3}-\frac{2s-18}{s^2+9}+\frac{24}{s^4}$ is

(a) $5t+2t^2-2\cos 3t$	(b) $5t+2t^2-2\cos 3t+18\sin 3t$	(c) $5t+2t^2-2\cos 3t+6\sin 3t$	(d) Otherwise

(8) The Laplace transform of the partial derivatives of the function $\frac{\partial^2 u}{\partial x^2}$ is

(a) $s^2\frac{d^2u_1(x)}{dx^2}-su_1(x)$	(b) $\frac{d^2u_1(x)}{dx^2}-su_1(x)$	(c) $\frac{d^2u_1(x)}{dx^2}+su_1(x)$	(d) Otherwise

(9) The Laplace transform on both sides of the differential equation $\frac{d^2x(t)}{dt^2}=-4x(t)$ is

(a) $s^2\bar{x}(s)-sx(0)-$ $-x'(0)=-4\bar{x}(s)$	(b) $s^2\bar{x}(s)-x(0)-$ $-sx'(0)=-4\bar{x}(s)$	(c) $s^2\bar{x}(s)-x(0)-$ $-x'(0)=-4\bar{x}(s)$	(d) Otherwise

(10) Inverse Laplace Transform of the function $\frac{10(s+4)}{(s+2)^2}$ is			
(a) $10(1-2t)e^{-2t}$	**(b)** $10(1+2t)e^{2t}$	**(c)** $20(1-2t)e^{-2t}$	(d) Otherwise
(11) The separation of variables method is used to solve			
(a)The ordinary differential equation	(b)The partial differential equation	(c) The non-linear partial differential equation	(d) Otherwise
(12) The solution $u(x,t)$ to the partial differential equation $\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$ is in the form			
(a) $Ce^{-\lambda^2\alpha^2 t}(A\cos\lambda t+B\sin\lambda t)$	(b) $Ce^{-\lambda^2\alpha^2 t}(A\cos\lambda x+B\sin\lambda x)$	(c) $Ce^{\lambda^2\alpha^2 t}(A\cos\lambda t+B\sin\lambda t)$	(d) Otherwise
(13) The solution $u(x,t)$ to the partial differential equation $\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$ is in the form			
(a) $Ce^{-\lambda^2\alpha^2 t}(A\cos\lambda x+B\sin\lambda x)$	(b) $(Ae^{\lambda t}+Be^{-\lambda t})x$ $x(C\cos\lambda x++D\sin\lambda x)$	(c) $(A\cos\lambda t+B\sin\lambda t)x$ $x(C\cos\lambda x++D\sin\lambda x)$	(d) Otherwise
(14) The solution $f(r)$ to the ordinary differential equation $r^2\frac{d^2 f(r)}{dr^2}+r\frac{df(r)}{dr}-\lambda^2 f(r)=0$ is in the form			
(a) $Cr^{\lambda}+Dr^{-\lambda}$	(b) $Cr^{\lambda^2}+Dr^{-\lambda^2}$	(c) $(A\cos\lambda r+B\sin\lambda r)$	(d) Otherwise
(15) Classification of partial differential equation $\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial y^2}=0$ is			
(a) elliptic-type	(b) parabolic-type	(c) Hyperbolic	(d) Otherwise
(16) Laplace operator $\nabla^2 u$ in spherica coordinates equal			
(a) $u_{rr}+\frac{1}{r}u_r+u_{zz}$	(b) $u_{rr}+\frac{1}{r}u_r+u_{rr}$	(c) $u_{rr}+\frac{1}{r}u_r-\frac{1}{r^2}u_{\theta\theta}$	(d) Otherwise
(17) Gamma funnction $\Gamma(s)$ is			
(a) $\int_0^\infty e^{-x}x^{s-1}dx$	(b) $\int_0^\infty e^{s-1}x^{-s}dx$	(c) $\int_0^\infty e^{-x}x^{p-1}dx$	(d) Otherwise
(18) Gamma funnction $\Gamma(s+1)$ is			
(a) $s\Gamma(s+1)$	(b) $(s+1)\Gamma(s)$	(c) $s^2\Gamma(s)$	(d) Otherwise
(19) $\int_0^\infty x^6 e^{-2x}dx$ is			

(a) 4.125	(b) 5.225	(c) 6.5	(d) Otherwise
(20) $\beta(\frac{1}{2},\frac{1}{2})$ equal			
(a) π	(b) $\sqrt{\pi}$	(c) $\sqrt{2\pi}$	(d) Otherwise

Questions ended with best wishes for success ,,,

Prof. A. M. Abd-Alla

First Semester Exam 2023

Sohag University

2nd year (chem. & Physics)
Code:207 Math
Time allowed: **2** hours

Title: General Mathematics (i)
Date: 18/ 01/ 2023
Degree: 100 degrees

Mathematics Department

Answer the following questions: (The exam in 3 pages)

Q1. **Choose the correct answer: (**Three degrees for each point**)**

(1) The domain of the function $f(x) = \ln(4 - x^2 - y^2)$ is located … the circle $x^2 + y^2 = 4$.

(a) *outside*, (b) *inside*, (c) *on*, (d) *otherwise*

(2) The range of the function $f(x, y) = \ln(2y - x + 2)$ is ….

(a) R^2 (b) R (c) $(0, \infty]$ (d) upper the line $y = \frac{1}{2}x - 2$

(3) The value of C such that the function $f(x,y) = \begin{cases} \frac{x^4 - y^4}{x^2 - y^2}, & y \neq x \\ C & y = x \end{cases}$ is continuous...

(a) xy (b) $-xy$ (c) $2\,x^2$ (d) *otherwise*

(4) The value of the $\lim\limits_{(x,y)\to(0,0)} \frac{2xy^2}{x^2 + y^2}$ is…

(a) 2 (b) -1 (c) 0 (d) *not exists*.

(5) The value of the $\lim\limits_{(x,y)\to(2,2)} \frac{2x^2 - 5xy + 3y^2}{x^2 + 3xy - 4y^2} = \cdots$

(a) $\frac{-1}{5}$ (b) -1 (c) xy^2 (d) *not exists*

(6) If $z = f(\frac{xy}{x^2 + y^2})$, then $\boldsymbol{x z_x + y z_y} = \cdots$

(a) 0 (b) -1 (c) $\frac{x}{y}$ (d) *otherwise*

(7) If $z = x^2 + y^2$, $x = \cos t$, $y = \sin t$, then $\frac{dz}{dt} = ...$

(a) 0 (b) -2 (c) $2 \sin t \cos t$ (d) otherwise

(8) If $w = x^2 \ln(y^2 + z^2)$, then $\frac{\partial w}{\partial x} = ...$

(a) $\frac{2x}{\ln(y^2 + z^2)}$ (b) $2x$ (c) $2y \ln(y^2 + z^2)$ (d) $2x \ln(y^2 + z^2)$

With my best wishes and warmest regards *Dr. E. El-Sanousy*

(9) The range of the function $f(x) = \sqrt{x^2 + y^2}$ is …

(a) R^2, (b) $(0, \infty)$, (c) $[0, \infty)$, (d) *otherwise*

(10) If $z = f(x, y)$, satisfies the equation $x\,y\,z = 1$, then $\dfrac{\partial z}{\partial x} = \ldots$

(a) $\dfrac{x}{y}$ (b) $-\dfrac{y}{x}$ (c) $-\dfrac{z}{x}$ (d) otherwise

(11) The differential equation $\dfrac{dx}{dy} - \dfrac{1}{y}x = y^3$ is …… equation.

(a) linear (b) exact (c) separable (d) otherwise

(12) The integrating factor of the differential equation $\dfrac{dx}{dy} - 3x = e^{2y}$ is ……

(a) $\mu = e^{-3x}$ (b) $\mu = e^{x}$ (c) $\mu = e^{-3y}$ (d) otherwise

(13) The degree of the differential equation $\dfrac{d^2y}{dx^2} + 2(\dfrac{dy}{dx})^2 = x^2 \ln(\dfrac{d^2y}{dx^2})$ is …….

(a) 1 (b) $\mu = 2$ (c) 4 (d) indefinite

(14) The differential equation $(x\,e^{y/x} - y)dx + xdy = 0$ is …… equation.

(a) homogeneous (b) exact (c) separable (d) otherwise

(15) The order of the differential equation $\dfrac{d^2y}{dx^2} + y = \tan(\dfrac{dy}{dx})$ is …….

(a) 1 (b) $\mu = 2$ (c) 3 (d) indefinite

(16) The differential equation of the family of curves $y = A\,e^{Bx}$ is …….

(a) $y'' - y' = 0$ (b) $yy'' - y' = 0$ (c) $y'' + yy' = 0$ (d) $yy'' - (y')^2 = 0$

(17) The solution of the exact differential equation $(x^2 + y^2)dy + 2xydx = 0$ is …….

(a) $2x^2y + \dfrac{1}{3}y^3 = c$ (b) $x^2y + y^2 = c$ (c) $x^2y + \dfrac{1}{3}y^3 = c$

(d) $x^2y - \dfrac{1}{3}y^3 = c$

(18) The solution of the differential equation $\dfrac{dy}{dx} = e^{x+y}$ is …….

(a) $-e^{-y} = e^{x} + c$ (b) $e^{-y} - e^{x} = c$ (c) $e^{y} = e^{x} + c$ (d) $e^{y} = e^{-x} + c$

(19) The integrating factor of the differential equation $x \ln x \dfrac{dy}{dx} + y = 2\ln x$ is …

(a) $ln\,(lnx)$ (b) $ln\,(x)$ (c) $\boldsymbol{e^x}$ (d) x

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(20) The solution of the differential equation $\dfrac{dy}{dx}-3y=e^{2x}$ is …….

(a) $y=-e^{2x}+ce^{3x}$ (b) $y=-e^{2x}+c$ (c) $y=e^{-x}+ce^{x}$ (d) $e^{y}=e^{2x}+c$

(21) The degree of differential equation $\dfrac{dy}{dx}=\dfrac{x^4-y^4}{(x^2-y^2)xy}$ is ……

(a) 1 (b) 2 (c) 3 (d) 4

(22) The order and degree of $(\dfrac{d^3y}{dx^3})^2+(\dfrac{d^2y}{dx^2})^3+y=0$ are 3 and 2 respectively.

(a) 3,2 (b) 2,3 (c) 3, not defined (d) none of these

(23) The solution of the differential equation $\dfrac{dy}{dx}=\dfrac{y}{x}$ is …….

(a) $y=cx$ (b) $y=e^{x}$ (c) $y=e^{-x}$ (d) $y+x=c$

(24) If $u=x^{y^2}$, then $\dfrac{u_y}{u_x}=$ ……….

(a) $\dfrac{x}{y}$ (b) $\dfrac{2x}{y}\ln x$ (c) $\dfrac{y}{x}$ (d) 2

(25) If $u=\cos^{-1}(\dfrac{x}{y})$, then $\dfrac{\partial u}{\partial x}=$ ……….

(a) $\sqrt{y^2-x^2}$ (b) $\sqrt{x^2-y^2}$ (c) $\dfrac{1}{\sqrt{x^2-y^2}}$ (d) $\dfrac{-1}{\sqrt{y^2-x^2}}$

Sohag University	شعار القسم الكلية حاصلة علي شهادة الاعتماد من الهيئة القومية لضمان جودة التعليم والاعتماد منذ ٢٠١٢/٧/١٢ م	Faculty of Science

Final Exam of Fields Theory (*Course Code: Math* 453)
for 4th level students– Mathematics program
1st Semester – Academic Year 2022/2023

Exam date: *Thursday: ١٩/01/2023* **Time:** ٣ *hours* **Total mark:** ١٦٠ *marks* **Pages:** 3	**Examiner:** *Aboelnour N. Abdalla* SP00QF140001

Answer the Following Questions:

(i) The Maxwell's equations for any materials are:

$$\vec{\nabla}\cdot\vec{E}=\frac{\rho}{\varepsilon_o}\quad(1),\quad \vec{\nabla}\cdot\vec{B}=0\quad(2),\quad \vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}\quad(3),\quad \vec{\nabla}\times\vec{B}=\mu_o(\vec{J}+\varepsilon_o\frac{\partial\vec{E}}{\partial t})\quad(4)$$

Answer the following questions using Maxwell's equations above.

(1) Equation No. (1) in Maxwell's previous equations is called the equation:			
(A) Faraday's Law	(B) Ampere's Law	(C) Gauss' Law	(D) Otherwise
(2) Equation No. (2) in Maxwell's previous equations is called the equation:			
(A) Gauss' Law	(B) Faraday's Law	(C) Ampere's Law	(D) Otherwise
(3) Equation No. (3) in Maxwell's previous equations is called the equation:			
(A) Faraday's Law	(B) Gauss' Law	(C) Newton's Law	(D) Otherwise
(4) Equation No. (4) in Maxwell's previous equations is called the equation:			
(A) Gauss' Law	(B) Faraday's Law	(C) Ampere's Law	(D) Otherwise
(5) To obtain Maxwell's previous equations in a vacuum, it is necessary to put:			
(A) $\rho \neq 0$ *and* $\vec{J} \neq 0$.	(B) $\rho = 0$ *and* $\vec{J}$ =0	(C) $\rho \neq 0$ *and* $\vec{J}$ =0.	(D) Otherwise
(6) The definition of the symbol $\vec{E}$ in Maxwell's previous equations is:			
(A) Electric charge	(B) The permittivity	(C) Electric field	(D) Otherwise
(7) The definition of the symbol μ_o in Maxwell's previous equations is:			
(A) Magnetic field	(B) Speed of light	(C) Electric charge	(D) Otherwise
8) The definition of the symbol $\vec{B}$ in Maxwell's previous equations is:			
(A) Magnetic field	(B) Electric charge	(c) The permittivity	(D) Otherwise
(9) The definition of the symbol $\vec{J}$ in Maxwell's previous equations is:			
(A) Velocity of light	(B) Electric current	(C) Magnetic field	(D) Otherwise
(10) The definition of the symbol ρ in Maxwell's previous equations is:			
(A) Speed of light	(B) Electric charge	(C) The permeability	(D) Otherwise
(11) The definition of the symbol ε_o in Maxwell's previous equations is:			
(A) Electric charge	(B) Magnetic field	(C) Speed of light	(D) Otherwise
(12) In the last two equations(3) and (4) , we see a changing magnetic field generates …………			
(A) Electric field	(B) Electric charge	(C) Magnetic field	(D) Otherwise
(13) Also in the last two equations (3) and (4), we see a changing electric field generates			
(A) Electric field	(B) Electric charge	(C) Magnetic field	(D) Otherwise

(14)	In a vacuum, from Maxwell's previous equations, the wave equation can be obtained in the following form:		
(A) $\vec{\nabla}^2 E = c \frac{\partial^2 \vec{E}}{\partial t^2}$	(B) $\vec{\nabla}^2 E = \mu_o \varepsilon_o \frac{\partial^2 \vec{E}}{\partial t^2}$	(C) $\vec{\nabla}^2 E = c^2 \frac{\partial^2 \vec{E}}{\partial t^2}$	(D) Otherwise
(15)	As the wave equation propagates at a speed equal to the speed of light, which is equal to:		
(A) $=3 \times 10^8\ cm/s$	(B) $= 3 \times 10^8\ m/s$	(C) $\approx 3 \times 10^8\ m/s$	(D) Otherwise
(16)	It is very important that the associated magnetic field also satisfies the wave equation in the form:		
(A) $\vec{\nabla}^2 B = c \frac{\partial^2 \vec{B}}{\partial t^2}$	(B) $\vec{\nabla}^2 B = \frac{1}{c} \frac{\partial^2 \vec{B}}{\partial t^2}$	(C) $\vec{\nabla}^2 B = \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2}$	(D) Otherwise
(17)	Maxwell's four equations with the Lorentz force contain all the knowledge of electrodynamics. There are many applications for this not including, for example:		
(A) Electronic devices	(B) Power generation	(C) Power supply	(D) Otherwise
(18)	The electric field $\vec{E}$ as a vector in three-dimensional space has: ……… components		
(A) Three	(B) Two	(C) Four	(D) Otherwise
(19)	The four Maxwell equations can be represented in .... different ways.		
(A) Four	(B) Three	(C) Two	(D) Otherwise
(20)	The forms of four Maxwell equations are useful and can be transformed into each other using …………… mathematical theorems.		
(A) Four	(B) Three	(C) Two	(D) Otherwise
(21)	The differential form of Maxwell's equations is useful for calculating the magnetic and electric fields at .... in space.		
(A) Mathematics	(B) An entire region	(C) A single point	(D) Otherwise
(22)	The integral form of Maxwell's equations is useful to compute the fields over .... in space.		
(A) An entire region	(B) A single point	(C) Physics	(D) Otherwise

(23)	The divergence integral theorem (Gauss integral theorem) may be written as: $\int_V (\vec{\nabla}.\vec{F}) = \cdots$		
(A) $\oint_A \vec{F} \cdot d\vec{a}$	(B) $\oint_A \vec{F} \times d\vec{a}$	(C) $\oint_A^B \vec{F} \cdot d\vec{a}$	(D) Otherwise
(24)	In the previous question No. (23) the symbol A represents a ………. containing any volume.		
(A) Area	(B) Surface	(C) Line	(D) Otherwise
(25)	In the previous question No. (23), the small circle around the integral indicates that this surface must satisfy …………….		
(A) No condition	(B) The surface is open	(C) The surface has a hole	(D) Otherwise
(26)	The second important theorem necessary for understanding the Maxwell's equations is $\int_A (\vec{\nabla} \times \vec{F}) \cdot d\vec{a} = \cdots$		
(A) $\oint_L \vec{F} \times d\vec{a}$	(B)) $\oint_L \vec{F} \cdot d\vec{a}$	(C)) $\oint_L \vec{F} \cdot d\vec{l}$	(D) Otherwise
(27)	The L represents …………… in space.		
(A) Surface	(B) Line	(C) Area	(D) Otherwise
(28)	The first Maxwell's equation in integral form is: $\oint_A \vec{E} \cdot d\vec{a} = \cdots\ \cdots\ \cdots$		
(A) $Q/_{\pi}$	(B) $Q/_{\varepsilon_o}$	(C)) $Q/_{\varepsilon}$	(D) Otherwise

(29)	Where the definition of Q in the previous line is: ……………		
(A) The total charge	(B) the charge density	(C) The electric charge	(D) Otherwise
(30)	The third Maxwell equation in integral form is: $\oint_L \vec{E} \cdot d\vec{l}$		
(A) $\int_A \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$	(B) $\int_A \frac{\partial \vec{B}}{\partial t} \times d\vec{a}$	(C) $\int_V \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$	(D) Otherwise
(31)	The fourth Maxwell equation in integral form is: $\mu_o \vec{I} + \mu_o \varepsilon_o \int_A \frac{\partial \vec{E}}{\partial t} \cdot d\vec{a} = \cdots\cdots\cdots$		
(A) $\int_V \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$	(B) $\oint_L \vec{E} \cdot d\vec{l}$	(C) $\oint_L \vec{B} \cdot d\vec{l}$	(D) Otherwise
(32)	The definition of $\vec{I}$ in the previous question (31) is:		
(A) the electric field	(B) the electric charge	(C) the **electric current**	(D) Otherwise
(33)	If the electric field $\vec{E}$ does not change over time, then the second summand in (31) is: $\cdots\cdots$		
(A) Zero	(B) Constant	(C) $\mu_o \varepsilon_o$	(D) Otherwise
(34)	The 2nd Maxwell equation **in integral form** is: $\cdots\cdots\cdots$		
(A)) $\oint_A \vec{E} \cdot d\vec{a} = 0$	(B) $\oint_A \vec{B} \cdot d\vec{a} = 0$	(C) $\oint_A \vec{B} \cdot d\vec{a} = {}^{Q}/_{\varepsilon_o}$	(D) Otherwise
(35)	The following integral form $\oint_L \vec{B} \cdot d\vec{l} = \mu_o I$ is called $\cdots\cdots\cdots$		
(A) Newton's Law	(B) Ampere's law	(C) Faraday's Law	(D) Otherwise
(36)	Which of the following represents a wave equation?		
(A) $\frac{\partial^2 V}{\partial t^2} = c^2 \nabla^2 V$	(B) $\nabla^2 V = 0$	(C) $\frac{\partial V}{\partial t} = k \nabla^2 V$	(D) Otherwise
(37)	The differential equation representing the heat equation is: …………		
(A) $\frac{\partial^2 V}{\partial t^2} = c^2 \nabla^2 V$	(B) $\nabla^2 V = 0$	(C) $\frac{\partial V}{\partial t} = k \nabla^2 V$	(D) Otherwise
(38)	$\frac{\partial^2 V}{\partial t^2} = c^2 \frac{\partial^2 V}{\partial x^2}$ represents the equation for:		
(A) Vibration of a string	(B) Heat flows a thin rod	(C) Motion of a projectile	(D) Otherwise
(39)	Gauss' Theorem (or the Divergence Theorem), may be written as: $\iiint_V (\vec{\nabla} \cdot \vec{A}) d\tau = \cdots\cdots\cdots\cdots$		
(A) $\oint_S \vec{A} \times d\vec{a}$	(B) $\oint_V \vec{A} \cdot d\vec{a}$	(C) $\oint_S \vec{A} \cdot d\vec{a}$	(D) Otherwise
(40)	Stokes' Theorem may be written as: $\oint_P \vec{A} \times d\vec{l}$		
(A) $\iint_S (\vec{\nabla} \times \vec{A}) \times d\vec{a}$	(B) $\iint_S (\vec{\nabla} \times \vec{A}) \cdot d\vec{a}$	(C) $\iint_S (\vec{\nabla} \times \vec{a}) \cdot d\vec{A}$	(D) Otherwise

SP00QF140001

Sohag University

الكلية حاصلة علي شهادة الاعتماد من الهيئة القومية

لضمان جودة التعليم والاعتماد منذ ٢٠١٢/٧/١٢ م

Faculty of Science

Final Exam of an elective course (Tensors) (*Course Code: Math 327*)
for 3[d] level students credit hours – Mathematics program
1-st Semester – Academic Year 2022/2023

Exam date: *Monday: 23/01/2023* **Time:** *2 hours* **Total mark:** *50 marks* **Pages:** *3*	**Examiner:** *Aboelnour N. Abdalla* SP00QF140001

Answer the Following Questions:

(1)	Einstein’s theory of relativity required a tensor of rank ……………..		
A) 1	(B) 2	(C) 0	(D) Otherwise
(2)	0^{th} rank tensor is called.		
(A) Scalar	(B) Vector	(C) Matrix	(D) Otherwise
(3)	1^{st} rank tensor is called ……….		
(A) Scalar	(B) Vector	(C) Matrix	(D) Otherwise
(4)	Which is the following quantity being scalar?		
(A) Velocity	(B) Acceleration	(C) Force	(D) Otherwise
(5)	Which is the following quantity being vector?		
(A) mass	(B) Velocity	(C) Pressure	(D) Otherwise
(6)	How many components in tensor of rank n in 3D?		
$(A)3^n$	(B) 6^n	(C) 4^n	(D) Otherwise
(7)	How many ranks in A_{ijk} ?		
(A)1	(B)2	(C)3	(D) Otherwise
8)	$\delta_{ij}\delta_{jk} = \cdots$		
(A) δ_{ik}	(B) δ_{ijk}	(C) δ_{ikj}	(D) Otherwise
(9)	A tensor δ_{ik} is said a symmetric if it satisfies the following:		
(A) $\delta_{ij} = \delta_{jk}$	(B) $\delta_{ji} = \delta_{jk}$	(C) $\delta_{ik} = \delta_{ki}$	(D) Otherwise
(10)	Kronecker delta is equal to 1 if:		
$(A)\ i \neq j$	$(B)\ i = j$	$(C)\ i < j$	(D) Otherwise
(11)	$\varepsilon_{kij}\varepsilon_{kim} =$………………………		
(A) $2\delta_{jm}$	(B) δ_{jm}	(C) zero	(D) Otherwise
(12)	$\varepsilon_{kij}\varepsilon_{kim} =$………………………		
(A) $2\delta_{jm}$	(B) 6	(C)zero	(D) Otherwise
(13)	$\delta_{ik}\varepsilon_{ijk} =$…………………………		
(A) δ_{ik}	(B) 4	(C) zero	(D) Otherwise
(14)	$\delta_{23}\delta_{22} =$………………………		
(A) 1	(B)2	(C)3	(D) Otherwise

(15)	$a_j\delta_{jk}$ =.............................		
(A) zero	(B) 1	(C)2	(D) Otherwise
(16)	If $i\ runs\ from$ 1 $to\ n$, then δ_{jj} =........................		
(A)zero	(B)1	(C) n	(D) Otherwise
(17)	Scalar product with Kronecker delta $\vec{a}.\vec{b} = \cdots$		
(A) $a_ib_j\delta_{ij}$	(B) $a_ib_k\delta_{ij}$	(C) $a_ib_i\delta_{ii}$	(D) Otherwise
(18)	$\in_{ijk}\in_{ilm}$ is a tensor in 3D of rank		
(A) 4	(B) 5	(C) 6	(D) Otherwise
(19)	$\in_{ijk}\in_{pqr}$ is a tensor in 3D of rank		
(A) 4	(B) 5	(C) 6	(D) Otherwise
(20)	$\in_{ijk}$ *is*		
(A) antisymmetric	(B) parallel	(c) symmetric	(D) Otherwise
(21)	$\in_{ijk} = \cdots$		
(A) $\in_{kji}$	(B) $-\in_{jik}$	(C) $\in_{jik}$	(D) Otherwise
(22)	$\varepsilon_{kij}\varepsilon_{klm} =$		
(A) $\delta_{il}\delta_{jm} - \delta_{im}\delta_{kl}$	(B) $\delta_{kl}\delta_{jm} - \delta_{im}\delta_{jl}$	(C) $\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$	(D) Otherwise
(23)	$[\vec{A} \times (\vec{B} \times \vec{C})] =$		
(A) $\varepsilon_{kij}\varepsilon_{klm}A_jB_lC_m$	(B) $\varepsilon_{kij}\varepsilon_{klm}A_jB_kC_k$	(C) $\varepsilon_{kij}\varepsilon_{klm}A_mB_lC_m$	(D) Otherwise
(24)	$\delta_{ik}\varepsilon_{ijk} =$		
(A) Zero	(B) ε_{ijk}	(C) ε_{klm}	(D) Otherwise
(25)	$B\delta_{33} =$		
(A) Zero	(B) ε_{33}	(C) B	(D) Otherwise
(26)	$b_j\delta_{jk}$ =.....................		
(A) Zero	(B) b_k	(C) δ_{jk}	(D) Otherwise
(27)	If $i\ runs\ from$ 1 $to\ m$, then δ_{jj} =.............		
(A) Zero	(B) m	(C) δ_{jk}	(D) Otherwise
(28)	Scalar product of two orthonormal vectors may be written as:		
(A) $\hat{e_i}\,.\,\hat{e_j} = 1$	(B) $\hat{e_i}\,.\,\hat{e_j} = 0$	(C) $\hat{e_i}\,.\,\hat{e_j} = \delta_{ij}$	(D) Otherwise
(29)	Scalar product with Kronecker delta may be written as:		
(A) $\vec{a}.\vec{b} = a_ib_j\delta_{ij}$	(B) $\vec{a}.\vec{b} = a_ib_j$	(C) $\vec{a}.\vec{b} = a_ib_j\delta_{jj}$	(D) Otherwise
(30)	$\vec{A}.\vec{B} \times \vec{c}$ =.................		
(A) $\sum_{ijk} A_iB_jC_k$	(B) $\sum_{ijk} \epsilon_{ijk}A_iB_jC_k$	(C) $A_iB_jC_k$	(D) Otherwise
(31)	$\varepsilon_{kij}\varepsilon_{klm}$ =.........................		
(A)$\varepsilon_{kij}\varepsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$	(B) $\varepsilon_{kij}\varepsilon_{klm} = \delta_{nl}\delta_{jm} - \delta_{im}\delta_{jl}$	(C) $\varepsilon_{kij}\varepsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{kl}$	(D) Otherwise
(32)	Which is the following quantity has only magnitude?		
(A) matrix	(B) tensor	(C) Vector	(D) Otherwise
(33)	In the dyad the direction is expressed as the sum of .... components		
(A) 1	(B) 2	(C) 3	(D) Otherwise
(34)	When $\in_{kji} = 1$ this case is called a		

(A) Normal		(B) Cyclic permutation	(C) Vector	(D) Otherwise
(35)	When $\in_{ijk}= 1$ this case is called a ………………….			
(A) Vector		(B) Cyclic permutation	(C) Normal	(D) Otherwise
(36)	Det A = …….............			
(A) $\in_{kij}\ a_{i1}a_{j2}a_{k3}$		(B) $\in_{jik}\ a_{i1}a_{j2}a_{k3}$	(C) $\in_{ijk}\ a_{i1}a_{j2}a_{k3}$	(D) Otherwise
(37)	Orthorhombic is a cube such with angles in three direction all angle equal ………….. degree			
(A) 45°		(B) 30°	(C) 90°	(D) Otherwise
(38)	A Tensor of such as A_{ijklm} where i, j, k, l, m run 1 to 4 has ……………….. components			
(C) 4^n		(B) 6^n	$(A)3^n$	(D) Otherwise
(39)	A Tensor of such as A_{ijklm} where i, j, k, l, m run 1 to 4 has ……………….. rank			
(A) 5		(B) 4	(C) 3	(D) Otherwise
(40)	The following equation $d\varphi = \frac{\partial\varphi}{\partial x^1}dx^1 + \frac{\partial\varphi}{\partial x^2}dx^2 + \cdots ... + \frac{\partial\varphi}{\partial x^N}dx^N$ may be written in tensorial form as: ………..			
(A) $d\varphi = \frac{\partial\varphi}{\partial x^N}dx^i$		(B) $d\varphi = \frac{\partial\varphi}{\partial x^i}dx^i$	(C) $d\varphi = \frac{\partial\varphi}{\partial x^i}dx^j$	(D) Otherwise

SP00QF140001

First Semester Exam June, 2023

4th year (Mathematics+Computer Science) **Title**: Numerical Analysis (ii)
Code:405 Math **Date**: 15/ 01/ 2023 **Time allowed**: **3** hours

Mathematics Department

Answer the following questions: (The exam in 4 pages)

Q1. **Choose the correct answer: (**4 degrees for each point**)**

1. The finite difference method with truncation error of order…
   $(a) O(h^2)$ $(b)\ O(h^3)$ $(c) O(h^4)$ (d) non of these
2. In Runge-Kutta fourth order method $k_2 = \ldots$..
   (a) $hf\ (x_0 + h, y_0 + k_1)$ (b) $hf(x_0 + \frac{h}{2}, y_0 + k_1)$
   (c) $h\ f(x_0 + \frac{h}{2}, y_0 + \frac{1}{2}k_1)$ (d) $hf(x_0 + \frac{h}{2}, y_0 + \frac{1}{2}k_2)$
3. In Runge-Kutta second order method if $\alpha_1 = 0, \alpha_2 = 1, \alpha = \beta = \frac{1}{2}$, then we have ……. mehod
   (a) Heun's (b) Midpoint (c) modified Euler (d) none of these

⇒ If $\frac{dy}{dx} = x^2 + 5,\ \ y(0) = 0,\ \ 0 \le x \le 1,\ h = 0.25$, by using Euler's method the value of

4. $y(0.25) = \ldots..$

   (a) 0.2552 (b) 1.2552 (c) 2.5417 (d) 3.8906

5. $y(0.50) = \ldots..$

   (a) 3.8906 (b) 2.2552 (c) 0.5417 (d) 2.5417

6. $y(0.75) = \ldots..$

   (a) 3.8906 (b) 0.2552 (c) 0.38906 (d) 3.2517

7. the boundary value problem has a unique solution when …..

   (a) $\frac{\partial f}{\partial y}(x, y, y') > 0\ \ and\ \ \frac{\partial f}{\partial y'}(x, y, y') > M$

   (b) $\frac{\partial f}{\partial y}(x, y, y') < 0\ \ and\ \ \frac{\partial f}{\partial y'}(x, y, y') \le M$

   (c) $\frac{\partial f}{\partial y}(x, y, y') > 0\ \ and\ \ \frac{\partial f}{\partial y'}(x, y, y') \le M$ (d) None of these

8. The second order Runga-kutta formula is

   (a) Euler's method (b) Newton's method

   (c) Modified euler's method (d) none

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⇒ If $\frac{dy}{dx} = x + y^2$, $y(0) = 0$, by Picard method iteration after first iteration,

9. $y(0.1) = ...$

(a) 0.0250 (b) 0.0050 (c) 0.0080 (d) 1.0352

10. $y(0.2) = ...$

(a) 0.250 (b) 0.0050 (c) 0.1020 (d) 0.020

11. If $f(x, y) = 1 + x(y)^2$ and D= $\{(x, y); 0 \le x \le 2, -1 \le y \le 1\}$ satisfies Lipschitz with Lipschitz constant L = ………..

(a) 2 (b) 3 (c) -1 (d) 4

⇒ If $\frac{dy}{dx} = e^x$, $y(0) = 1, h = 0.5,\ 0 \le x \le 1$, then, by using Midpoint method, we have

12. $i = 0$ *and* $k_2 = \cdots$

(a) 0.6420 (b) 1.6420 (c) 2.0642 (d) 0.0642

13. $i = 0$ and $y(0.5) = \cdots$

(a) 0.6420 (b) 1.6487 (c) 1.0642 (d) 2.0642

14. $i = 1$ and $y(1.0) = \cdots$

(a) 1.7183 (b) 0.6487 (c) 2.7183 (d) 2.0642

15. Fit the straight line to the following data.

X	1	2	3	4	5
y	1	2	3	5	5

(a) $y = x$ (b) $y = x + 1$ (c) $y = 2x$ (d) $y = 2x + 1$

16. If the equation $y = a\, e^{b\,x}$ can be written in linear form $Y = A + B\,X$, what are Y, X, A, B?

(a) $Y = logy,\ A = loga,\ B = b\ and\ X = x$ (b) $Y = y, A = a,\ B = b\ and\ X = x$
(c) $Y = y, A = a, B = logb\ and\ X = logx$ (d) $Y = logy, A = a, B = logb\ and\ X = x$

⇒ For data given below find ∑xi2yi .

X	0	1	2	3	4
Y	1	0	3	10	21

17. $\sum x_i^2 y_i = ...$

(a) 435 (b) 436 (c) 437 (d) 438

18. If the normal equations for a straight line $y = ax + b$ are $26 = 4a + 6b$ and

$34 = 6a + 4b$ then fit the above straight line is

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(a) $y = 5x - b$ (b) $y = 5x + b$ (c) $y = x + 5b$ (d) $y = x - 5b$

19. The first two steps of the fourth-order Runge-Kutta method use……

(a) Euler methods (b) Forward Euler method

(c) Backward Euler method (d) Explicit Euler method

⟹ A natural cubic spline S on $[0,2]$ is defined by

$$S(x) = \begin{cases} S_0(x) = 1 + 2x - x^3, & \text{if } 0 \le x < 1, \\ S_1(x) = 2 + b(x-1) + c(x-1)^2 + d(x-1)^3, & \text{if } 1 \le x \le 2. \end{cases}$$

Then,

20. $b = ...$

(a) -3 (b) 1 (c) -1 (d) None of these

21. $c = ...$

(a) -3 (b) 1 (c) -1 (d) None of these

22. $d = ...$

(a) -3 (b) 2 (c) -2 (d) None of these

23. Rewrite the differential equation $y'' - 2\,y' + 4y = t\,e^{2t}$ as a system of two first-order differential equations as follows

(a) $u_1' = u_2',\ u_2' = -4u_1 + 2u_2 + t\,e^{2t}$ (b) $u_1' = u_2,\ u_2' = -4u_1 + 2u_2 + t\,e^{2t}$

(c) $u_1' = u_2,\ u_2' = -4u_1 + u_2 + t$ (d) $u_1' = u_2,\ u_2' = 2u_2 + t\,e^{2t}$

24. Which is correct in shooting method to convert BVP into IVP

(a) $y = u_1$ and $y' = u_2$ (b) $y'' = u_2'$

(c) Both a and b (d) $y = u_2$ and $y' = u_2$

25. In Heun's method of Runge-Kutta of order 2 Methods the formula of $y_{i+1} = ...$

(a) $y_i + \frac{1}{4}k_1 + \frac{3}{4}k_2$ (b) $y_i + \frac{1}{2}(k_1 + k_2)$ (c) $y_i + k_2$ (d) $y_i + k_1 + k_2$

Q2. Put (✓) or (X) signs in as appropriate? [4 degrees for each point]

(1) When the free boundary conditions occur, the spline is called a natural spline ? ( )

(2) The local truncation error of the Runge-Kutta fourth method is $O(h^4)$. ( )

(3) In midpoint method $\alpha = 1$. ( )

(4) For first-degree splines, the slope of the spline may change abruptly at the knots? ( )

(5) The local error of the Modified Euler method is $O(h^3)$. ( )

(6) In Runge kutta second order method if $\alpha 2 = 1/4\ , \beta 2 = 3/4\ , \alpha = \beta = 2/3$, then we have Modified Euler method. ( )

(7) The shooting method is used to transformation IVP to BVP. ( )

(8) In cubic spline interpolation, the first and the second derivatives of the splines are

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continuous at the interior data points. ( )

(9) If $y' = x + y,\ y(0) = 1$, by using Picard method, after second approximation

$$y_2 = 1 + x + x^2 + \frac{x^3}{6}$$ ( )

⇒ Apply Runge-Kutta fourth order method to approximate value of y when $h = 0.2$ given that $y' = x + y,\ y(0) = 1$, then

(10) $k_1 = 0.2000$ ( )

(11) $k_2 = 2.2000$ ( )

(12) $k_3 = 0.2440$ ( )

(13) $k_4 = 1.4288$ ( )

(14) $y = 1.2428$ ( )

(15) The boundary value problem $y'' + e^{-xy} + \sin y' = 0,\quad 1 \le x \le 2,\quad y(1) = y(2) = 0$ has not a unique solution. ( )

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